



THE NATIONAL TECHNICAL UNIVERSITY OF UKRAINE  
"Igor Sikorsky Kyiv Polytechnic Institute"

Emblem  
departments  
(if  
available)

Department of Mathematical  
Physics and Differential  
Equations

## NAME OF THE COURSE

**Higher mathematics. Part 2. Integral calculus. Differential equations**

### Working program of the academic discipline (Syllabus)

#### Details of the academic discipline

<b>Level of higher education</b>	<i>First ((bachelor)</i>					
<b>Branch of knowledge</b>	<i>17 Electronics, automation and electronic communications</i>					
<b>Specialty</b>	<i>174 Automation, computer-integrated technologies and robotics</i>					
<b>Educational program</b>	<i>Technical and software automation tools</i>					
<b>Status of Discipline</b>	<i>Normative</i>					
<b>Form of education</b>	<i>Full-time</i>					
<b>Year of training, semester</b>	<i>First year, spring semester</i>					
<b>Scope of the discipline</b>	<i>210/ 7 credits</i>					
				Laboratory classes (computer practice)	Individual classes	Self-study work of student
	<b>Hours</b>	<b>54</b>	<b>54</b>	<b>0</b>	<b>0</b>	<b>102</b>
<b>Semester control/control measures</b>	Exam	Test	Modular control work (specify quantity)	Calculation graphic work, calculation work (specify quantity)	Home control work (specify quantity)	Abstract (specify quantity)
	+	-	1	0	1	0
<b>Lessons schedule</b>	<i>On the website of the university, as well as the website of the Faculty of Chemical Engineering</i>					
<b>Language of teaching</b>	<i>English</i>					

<b>Information about head of the course / teachers</b>	<p>Lecturer: Borysenko Olga Volodymyrivna, Ph.D., associate professor of the Department of Mathematical Physics and Differential Equations, Faculty of Physics and Mathematics  <a href="mailto:oborisenko1373@gmail.com">oborisenko1373@gmail.com</a>  <a href="https://intellect.kpi.ua/profile/bov114">https://intellect.kpi.ua/profile/bov114</a>  ORCID: <a href="http://orcid.org/0000-0002-2099-2911">http://orcid.org/0000-0002-2099-2911</a></p> <p>Practical: Borysenko Olga Volodymyrivna, Ph.D., associate professor of the Department of Mathematical Physics and Differential Equations, Faculty of Physics and Mathematics  <a href="mailto:oborisenko1373@gmail.com">oborisenko1373@gmail.com</a>  <a href="https://intellect.kpi.ua/profile/bov114">https://intellect.kpi.ua/profile/bov114</a></p> <p>Dyuzhenkova Olga Yuriivna, Ph.D., associate professor of the Department of Mathematical Physics and Differential Equations, Faculty of Physics and Mathematics  <a href="mailto:olgaduzen@gmail.com">olgaduzen@gmail.com</a>  ORCID: <a href="https://orcid.org/0000-0002-8146-0134">https://orcid.org/0000-0002-8146-0134</a></p>
<b>Placement of the course</b>	Website of the department, information resources in the library, electronic campus.

### Program of educational discipline

#### Description of the educational discipline, its purpose, subject of study and learning outcomes

The goal of the educational discipline is the formation of students' integral competence — the ability to think logically, the formation of students' personalities; development of their intelligence and abilities; the ability to solve complex specialized tasks and practical problems, characterized by complexity and uncertainty of conditions, during professional activities in the field of automation, computer-integrated technologies and robotics or in the learning process; use methods of mathematical analysis in engineering calculations.

Software competencies:

#### General competences (GC)

**GC 1.** Ability to think abstractly.

**GC 2.** Ability to apply knowledge of mathematics, to the extent necessary for the use of mathematical methods for the analysis and synthesis of automation systems.

**GC 3.** Ability to apply methods of system analysis, mathematical modeling, identification, and numerical methods to develop mathematical models of individual elements and automation systems as a whole, to analyze the quality of their functioning using the latest computer technologies.

#### Program learning outcomes (LO)

**LO 1.** Know linear and vector algebra, differential and integral calculus, functions of many variables, functional series, differential equations for functions of one and many variables, operational calculus, theory of functions of a complex variable, theory of probabilities and mathematical statistics, theory of random processes to the extent necessary for use of mathematical apparatus and methods in the field of automation.

**LO 2.** To be able to apply the methods of system analysis, modeling, identification and numerical methods to develop mathematical and simulation models of individual elements and automation systems as a whole, to analyze the quality of their functioning using the latest computer technologies.

### **Pre-requisites and post-requisites of the discipline (place in the structural and logical scheme of training according to the relevant educational program)**

It is taught in the first semester on the basis of full secondary or secondary professional education. In the structural and logical scheme of the training program in this direction, the academic discipline "Higher Mathematics" precedes and provides the following academic disciplines: Physics, Chemistry, Engineering and computer graphics, Mathematical methods in automation tasks, Computer modeling of processes and systems, Discrete mathematics and basics of system analysis, Theory of automatic control, Robotics.

### **Content of the academic discipline**

- 1. Differential calculus of functions of several variables. Limits, partial derivatives and differentials of functions of several variables. Extrema of a function of two variables.*
- 2. Integral calculus of a function of one variable. Complex numbers. Indefinite integral. Basic methods of integration. The definite integral. Improper integrals. Application of the definite integral.*
- 3. Ordinary differential equations. Differential equations of the first order. Differential equations of higher orders. Systems of differential equations.*

### **Educational materials and resources**

#### **Basic literature**

1. Дубовик В.П. Вища математика: навч. посіб. / Дубовик В.П., Юрик І.І. – К.: А.С.К., 2005. – 648 с.
2. Грималюк В.П. Вища математика: У 2 ч.: навч. посіб. / Грималюк В.П., Кухарчук М.М., Ясінський В.В. – К.: Віпол, 2004. – Ч. 1. – 376 с.
3. Грималюк В.П. Вища математика: У 2 ч.: навч. посіб. / Грималюк В.П., Кухарчук М.М., Ясінський В.В. – К.: Віпол, 2004. – Ч. 2. – 400 с.
4. Дубовик В.П. Вища математика. Збірник задач: навч. посіб. / Дубовик В.П., Юрик І.І. – К.: А.С.К., 2005. – 648 с.
5. В. С. Герасимчук, Г. С. Васильченко, В. І. Кравцов. Вища математика. Повний курс у прикладах і задачах: навч. посіб. [ Ч.1 ]. Лінійна й векторна алгебра. Аналітична геометрія. Вступ до математичного аналізу. Диференціальне числення функцій однієї та багатьох змінних. Прикладні задачі / - К.: Книги України ЛТД, 2009. - 578 с.
6. Герасимчук, В. С. Вища математика. Повний курс у прикладах і задачах: навч. посіб. [ Ч.2 ]. Невизначений, визначений та невластні інтеграли. Звичайні диференціальні рівняння. Прикладні задачі / В. С. Герасимчук, Г. С. Васильченко, В. І. Кравцов. - К. : Книги України ЛТД, 2010. – 470.
7. Авдеєва Т.В. Інтегральне числення функції однієї змінної. Навчальний посібник [Електронний ресурс] /КПІ ім. Ігоря Сікорського; Т.В. Авдеєва, О.Ю.Дюженкова, В.В. Листопадава. – Київ : КПІ ім. Ігоря Сікорського , 2023. – 151 с.  
<https://ela.kpi.ua/handle/123456789/56440>

#### **Additional literature**

1. Шкіль М.І., Колесник Т.В. Вища математика. - К.: Вища школа, 1986. – 512 с.

2. Стрижак Т.Г. Математичний аналіз: приклади і задачі: навч. посіб. / Стрижак Т.Г., Коновалова Н.Р. – К.: Либідь, 1995. – 240 с.
3. Клепко В. Ю. Вища математика в прикладах і задачах: навчальний посібник / В.Ю.Клепко, В.Л. Голець. – К.: Центр навчальної літератури, 2017. – 594 с
4. Зайцев Є. П. Вища математика: інтегральне числення функцій однієї та багатьох змінних, звичайні диференціальні рівняння, ряди: навч. посіб. / Є. П. Зайцев. – К.: Алерта, 2018. – 608 с.

## Educational content

### Methods of mastering an educational discipline (educational component)

#### 5.1. Didactic materials:

In lecture classes – Lecture (electronic version), explanation, brainstorming, problem tasks.

#### List of lectures

*Lecture 1. Limits, partial derivatives and differentials of functions of several variables*

- 1.1. Functions of several variables . Definitions, basic concepts.
- 1.2. The boundary and continuity of the Functions of several variables.
- 1.3. Partial derivatives, differentials.
- 1.4. Repeated differentiation.

*Lecture 2. Full differential. Differentials of higher orders. The derivative of a composite function. The derivative of an implicitly given Functions of several variables*

- 2.1. Complete differential and its application to approximate calculations.
- 2.2. Differentials of higher orders.
- 2.3. The derivative of a composite function.
- 2.4. The derivative of an implicitly given Functions of several variables .

*Lecture 3. Tangent plane and normal to the surface. Derivative in direction. Gradient*

- 3.1. Tangent plane and normal to the surface.
- 3.2. Scalar field.
- 3.3. Derivative in direction.
- 3.4. Gradient.
- 3.5. Taylor's formula.

*Lecture 4. Extrema of a function of two variables. Conditional extremum. The largest and smallest value of a function of many variables*

- 4.1. Necessary and sufficient conditions for the existence of an extremum of a function of two variables.
- 4.2. The concept of a conditional extremum. Lagrange function.
- 4.3. The largest and smallest value of a function of several variables.

*Lecture 5. Elements of higher algebra*

- 1.1. Complex numbers.
  - 5.1.1. Operations on complex numbers in algebraic form.
  - 5.1.2. Trigonometric form of writing complex numbers.
  - 5.1.3. Complex numbers in exponential form.
- 1.2. Algebra of polynomials.

*Lecture 6. Integral calculus of a function of one variable. Indefinite integral*

- 6.1. Primordial, its properties.
- 6.2. The indefinite integral, its properties.
- 6.3. Table of basic integrals.

*Lecture 7. Basic methods of integrating the indefinite integral*

- 7.1. Variable substitution method.
- 7.2. Bringing up under the sign of the differential.
- 7.3. Integration by parts.

*Lecture 8. Integration of expressions that depend on a quadratic trinomial. Integration of rational functions*

- 8.1. Integration of expressions that depend on a quadratic trinomial.
- 8.2. Integration of rational functions.
  - 8.2.1. Decomposition of a fractional-rational function into the sum of the simplest elementary rational functions.
  - 8.2.2. Integration of elementary fractions of types 1-4.

*Lecture 9. Integration of rational functions*

- 9.1. Integration of rational functions.
- 9.2. Examples of integrations of rational fractions.

*Lecture 10. Integration of trigonometric functions. Universal trigonometric substitution*

- 10.1. Integration of trigonometric functions using basic trigonometric formulas.
- 10.2. Universal trigonometric substitution.
- 10.3. Substituting a variable.

*Lecture 11. Integration of irrational functions. Trigonometric substitutions*

- 11.1. Integration of irrational expressions by the method of variable substitution.
- 11.2. Trigonometric substitutions.

*Lecture 12. Integration of the differential binomial*

- 12.1. Chebyshev's theorem.
- 12.2. Integrals that are not expressed in elementary functions.

*Lecture 13. The definite integral*

- 13.1. Definition, existence conditions of the definite integral, geometric meaning.
- 13.2. Newton-Leibnitz theorem.
- 13.3. Properties of the definite integral.
- 13.4. Integral with variable upper bound.

*Lecture 14. Calculation of the definite integral*

- 14.1. Integration by parts in a definite integral.
- 14.2. Variable substitution method.
- 14.3. Calculation of definite integrals over a symmetric segment of integration from even and non-even functions.

*Lecture 15. Improper integrals*

- 15.1. Improper integrals of the 1st kind.
- 15.2. Improper integrals of the 2nd kind.

*Lecture 16. Geometric and physical applications of the definite integral*

- 16.1. Application of the definite integral to the calculation of areas of region.
- 16.2. The length of the arc of the curve.

*Lecture 17. Geometric and physical applications of the definite integral*

- 17.1. Volume of a solid of revolution .
- 17.2. Surface of a solid of revolution.
- 17.3. Physical applications of the definite integral.

*Lecture 18. Ordinary differential equations. Differential equations of the first order. Cauchy's problem*

- 18.1. Basic definitions and concepts.
- 18.2. Existence theorem of a solution to the Cauchy problem.
- 18.3. Differential equations with separable variables.

*Lecture 19. Homogeneous differential equations of the first order. Equations that reduce to homogeneous ones*

- 19.1. Homogeneous differential equations of the first order.
- 19.2. Equations that reduce to homogeneous ones.
- Lecture 20. Linear differential equations of the first order. Bernoulli's equation*
  - 20.1. Linear differential equations of the first order.
    - 20.1.1. The Euler-Bernoulli method.
    - 20.1.2. The method of variation of an arbitrary constant.
  - 20.2. Bernoulli's equation.
  - 20.3. Equations in complete differentials.
- Lecture 21. Differential equations of higher orders. Equations that allow reduction of order*
  - 21.1. Basic concepts and definitions. Cauchy's problem.
  - 21.2. Equations that are integrated in quadratures.
  - 21.3. Equations that allow reduction of order.
- Lecture 22. Linear differential equations of higher orders*
  - 22.1. Basic concepts and definitions.
  - 22.2. Linear differential operator and its properties.
  - 22.3. Linear homogeneous differential equations of the second order (LHDE), properties of their solutions.
  - 22.4. Vronskian, its properties.
  - 22.5. Theorem on the structure of the general solution of a homogeneous equation.
- Lecture 23. Linear inhomogeneous differential equations (LIDE) of the second order*
  - 23.1. The structure of the general solution of the inhomogeneous equation.
  - 23.2. Superposition of solutions.
  - 23.3. Method of variation of arbitrary constants.
- Lecture 24. Linear homogeneous differential equations with constant coefficients of higher orders*
  - 24.1. Linear homogeneous differential equations of the second order with constant coefficients. Characteristic equation.
  - 24.2. Linear homogeneous differential equations of higher orders.
- Lecture 25. Linear inhomogeneous differential equations of higher orders with constant coefficients and a special right-hand side*
  - 25.1. Linear inhomogeneous differential equations of the second order with constant coefficients and a special right-hand side.
  - 25.2. Linear inhomogeneous differential equations of the nth order with constant coefficients and a special right-hand side.
- Lecture 26. Systems of linear differential equations*
  - 26.1. Normal systems of equations.
  - 26.2. Systems of linear differential equations with constant coefficients.
- Lecture 27. Application of differential equations*

### ***List of (approximate) practical classes***

*Practical lesson 1.* Derivatives and differentials of functions of many variables. Repeated differentiation.

*Practical lesson 2.* Differentiation of complex and implicit functions of many variables.

*Practical lesson 3.* Tangent plane and normal to the surface. Derivative in direction. Gradient.

*Practical lesson 4.* The extremum of a function of many variables. Conditional extremum. The largest and smallest value of a function of many variables.

*Practical lesson 5.* Modular short-term control work -1 "Differential calculus of functions of many variables". Integration using a table of integrals.

*Practical lesson 6.* The method of changing the variable, subsuming under the sign of the differential.

- Practical lesson 7.* Integration by parts.
- Practical lesson 8.* Integration of expressions that contain quadratic trinomials.
- Practical lesson 9, 10.* Integration of rational fractions.
- Practical lesson 11.* Integration of trigonometric functions.
- Practical lesson 12.* Integration of irrational expressions. Trigonometric substitutions.
- Practical lesson 13.* Chebyshev's theorem.
- Practical lesson 14.* Newton-Leibniz formula. Changing the variable in the definite integral.
- Practical lesson 15.* Calculation of definite integrals.
- Practical lesson 16.* Improper integrals of the first kind.
- Practical lesson 17.* Improper integrals of the second kind.
- Practical lesson 18.* Calculation of the area of a flat figure. Calculation of the length of the arc of the curve.
- Practical lesson 19.* Calculation of volumes of bodies, surface area of rotation.
- Practical lesson 20.* Modular short-term control work -2 "Integral calculus of a function of one variable". Differential equations of the first order with separated variables.
- Practical lesson 21.* Homogeneous differential equations and equations that reduce to homogeneous ones.
- Practical lesson 22.* Linear differential equations of the first order. Bernoulli's equation.
- Practical lesson 23.* Equations of higher orders that allow a decrease in order.
- Practical lesson 24.* Linear homogeneous equations with constant coefficients.
- Practical lesson 25.* Linear inhomogeneous equations with constant coefficients with a special right-hand side.
- Practical lesson 26.* Linear inhomogeneous equations with constant coefficients. Lagrange's method.
- Practical lesson 27.* Modular short-term control work – 3 on the topic "Ordinary differential equations". Application of differential equations.

In practical classes - Tasks to be completed (according to the specified list of basic literature).

**5.2. Technical support:** Microsoft Office Word, any software for the execution of graphic material (at the request of the student).

### **6. Self-study work of student**

Types of independent work - study of lecture material, preparation for classroom classes, problem solving, homework and calculation work (divided into two parts according to the semester plan certifications).

## **Policies and control**

### **7. Policies of academic discipline (educational component)**

Compliance with the provisions of the " Code of Honor of the National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute» (chapters 2 and 3). Read more: <https://kpi.ua/code>. Student cooperation in solving problem tasks is allowed, but each student defends the answers independently. Student interaction during the exam is strictly prohibited and any such activity will be considered a breach of academic integrity in accordance with the university's principles of academic integrity.

### **8. Types of control and rating system for evaluating learning outcomes (RS)**

**Current control:** express survey, survey on the subject of the lesson, writing MCW.

**Calendar control:** is held twice a semester as a monitoring of the current state of fulfillment of the syllabus requirements.

**Semester control:** exam.

**Terms of admission to semester control:** a minimum positive grade for MCW, credit for homework, semester rating of at least 36 points.

The student's rating in the discipline for the 1st semester consists of the points he receives for:

- 1) Eight answers (per student on average) in practical classes (provided that 7 students are interviewed in one lesson with a maximum group size of 25 people.
- 2) one modular control work;
- 3) one homework test;
- 4) the answer to the exam.

Work in practical classes

Under the condition of good preparation and active work in the practical session, the student receives 1 point. One or two best students in each practical session can be given 1 point as an incentive. The maximum number of points for all practical classes is equal to 1 point x 8 = 8 points.

Final control

One modular control work (MCW) is divided into three parts:

Modular short-term control work-1 "Differential calculus of functions of several variables": weighted score – 10 points;

Modular short-term control work-2 "Integral calculus of a function of one variable": weighted score – 20 points;

Modular short-term control work-3 "Ordinary differential equations": weighted score – 10 points.

(It is allowed to divide the Modular short-term control work into several thematic control papers that have the same total weighting point).

The maximum number of points for all test papers is equal to 40 points.

Home control work

Weight score – 12 points. The work is evaluated in terms of the percentage of correctly solved tasks.

Penalty and incentive points for:

- untimely (later than a week) submission of homework control work ..... – 2 points (for each week late);
- failure to perform homework and independent work ..... – 1 point (for each task);
- prizes in faculty and institute Olympiads in higher mathematics; preparation of scientific articles and reports for mathematical scientific and practical conferences; performance of tasks to improve didactic materials from the credit module..... + 6 points.



*Calculation of the rating scale (R):*

The sum of the weighted points of control measures during the semester is:

$$R_S = 40 + 20 = 60 \text{ points}$$

The examination component of the scale is equal to 40% of R, namely:

$$R_E = 40 \text{ points}$$

*Thus, the rating scale for the discipline is:*

$$R = R_S + R_E = 100 \text{ points.}$$

The size of the rating scale  $R=100$  points.

Size of the starting scale  $R_S = 60$  points.

The size of the examination scale  $R_E=40$  points.

At the decision of the examiner, without an additional survey, it is possible to assign (with the student's consent) a grade of "good" ("B" or "C" in the ECTS system) in the event that the student's starting rating is at least 0.9 of the maximum possible ( $R_S$ ), that is, at  $R_S \geq 54$  points.

Table of correspondence of rating points to grades on the university scale:

<i>PERSONAL SCORE</i>	<i>GRADE</i>
100-95	Excellent
94-85	Very Good
84-75	Good
74-65	Satisfactory
64-60	Sufficiently
Personal score < 60	Unsatisfactory
Personal score < 36 or the individual tasks is not credited	Not allowed to pass exam

**9. Additional information on the discipline (educational component)**

- *the possibility of enrolling in certificates of completion of distance or online courses on the relevant subject.*

**Working program of the academic discipline (syllabus):**

**Compiled**

Valentyna Lystopadova, Ph.D., Associate Professor

**Approved:** Department of Mathematical Physics and Differential Equations (Protocol №9 dated June 26, 2024)

**Agreed:** Methodical Commission of the Faculty of Chemical Engineering (Protocol №11 dated June 28, 2024)